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The output power of the second-harmonic-generation light derived from Fibonacci-class ferroelectric domains

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Abstract. On the basis of our studies on a method of constructing a class of one-dimensional Fibonacci-class quasilattice ferroelectric domain systems and the properties of the electric field of the second-harmonic-generation light, we obtained the output power per unit area of the second-harmonic-generation light and the efficiency of the output power versus input power. By means of the formula for the output second-harmonic-generation light, we have also obtained the analytical rules governing the absence effect for output ordinary-light spectral lines and that of extraordinary light.

1. Introduction

Since the important discovery by Shechtman *et al* [1] of the diffraction pattern with fivefold symmetry, much attention has been paid to the investigation of the optical properties of quasilattices and multilayer systems arranged following quasiperiodic orderings. In 1985, Merlin *et al* [2] first reported the realization of a quasiperiodic superlattice grown by molecular-beam epitaxy. Afterwards, Tamura and Wolfe [3] studied acoustic-phonon transmission through a realistic Fibonacci superlattice theoretically and obtained some results for the transmission spectra. Zhu and Ming [4] studied the properties of the second-harmonic-generation light (SHGL) spectral lines of a Fibonacci optical superlattice which was made from a single $LiNbO_3$ crystal with quasiperiodic laminar ferroelectric domain structures. Zhu *et al* [5] computed and measured the intensity of the SHGL for a $LiTaO_3$ crystal. The splitting rules for the spectra of two-dimensional (2D) Fibonacci quasilattices were studied in detail by us [6] recently.

In 1987, Kohmoto *et al* [7] studied the transmission of light through dielectric multilayers arranged following a one-dimensional (1D) Fibonacci chain (i.e. FC(1)). Gellermann *et al* [8] measured the optical transmission of quasiperiodic dielectric multilayer stacks of SiO_2 and TiO_2 thin films which were ordered according to a Fibonacci sequence. Huang *et al* [9] obtained theoretical results for the case of multilayers constructed following an intergrowth quasiperiodic sequence (i.e. FC(2)). We [10] have recently studied the general situation for the Fibonacci-class quasilattices (FC(n)) in detail and have found a very useful switchlike property of the transmission coefficient. FC(n) models, which are more general than the Fibonacci 1D chain, are 1D quasiperiodic models of a certain kind and were proposed by Fu *et al* [11]

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in 1997. Fu *et al* proved that the electronic energy spectra of FC(n) models were perfectly self-similar.

Since they form the theoretical basis for the application of multiwavelength SHGL devices to the frequency transmission of the harmonic generator, the output power per unit area of the SHGL and the efficiency of the output power versus input power are naturally very interesting problems to study. On the other hand, the absence effect for output spectral lines is also very important.

In this paper we obtain FC(n) by an indirect projection method from a 2D square lattice, and the position of the Nth atom on the projection line from the origin is given by

$$x_N = N + \frac{1}{\varphi_n}\left\lfloor \frac{N}{\varphi_n} \right\rfloor \tag{1}$$

where $\lfloor$ and $\rfloor$ represent the greatest integer function, and where

$$\varphi_n = \frac{1}{\sigma_n} = \frac{\sqrt{n^2+4}+n}{2} \tag{2}$$

is an irrational quadratic number and the positive root of the characteristic equation

$$x^2 - nx - 1 = 0. \tag{3}$$

Then the atoms in the chain FC(n) will arrange according to the following substitution rules:

$$\begin{aligned} S_1 &= \mathrm{B} \\ S_2 &= \mathrm{B}^{n-1}\mathrm{A} \\ S_j &= S_{j-1}^n S_{j-2} \qquad (j \geqslant 3). \end{aligned} \tag{4}$$

2. The output power of the SHGL

In order to manufacture the $LiNbO_3$ sample easily and compute the characteristic of the system conveniently, we made the two ferroelectric domains A and B each composed of two layers with different properties; these are shown in figure 1. We set [12]

$$\begin{aligned} &l_{\mathrm{A}}/l_{\mathrm{B}} = \varphi_n \\ &l_{\mathrm{A1}} = l \qquad l_{\mathrm{A2}} = l(1+\eta) \\ &l_{\mathrm{B1}} = l \qquad l_{\mathrm{B2}} = l(1-\varphi_n\eta) \end{aligned} \tag{5}$$

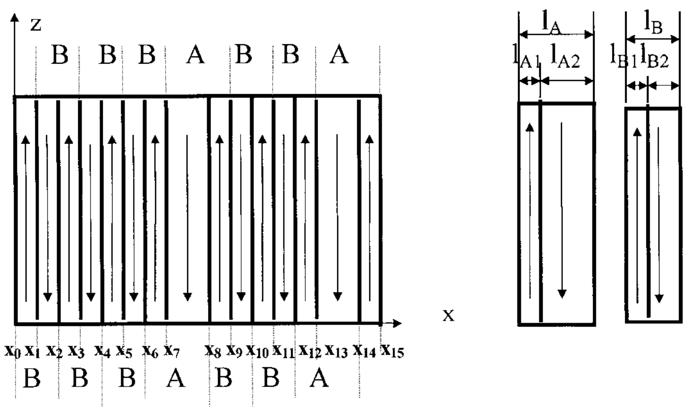

Figure 1. The construction of the FC(3) ferroelectric multilayer system.

where l is the structure parameter, and where η satisfies

$$\eta = \frac{2(\varphi_n - 1)}{1 + \varphi_n^2}. \tag{6}$$

Generally, η is responsible for the arrangement being periodic (when $\eta = 0$) or quasiperiodic (when $\eta \neq 0$).

In the case of SHGL with a single laser beam incident onto the surface of the FC(n) ferroelectric, the electric field of the fundamental beam, E_1, and that of the SHGL, E_2, satisfy the wave equation under the small-signal approximation [4, 13]:

$$\frac{\mathrm{d}E_2(x)}{\mathrm{d}x} = \frac{-\mathrm{i}\,32\pi\omega^2}{k^{2\omega}c^2} d(x) E_1^2 \mathrm{e}^{\mathrm{i}(k^{2\omega} - 2k^{\omega})x} \tag{7}$$

where

$$d(x) = \begin{cases} d_{33} & \text{in positive ferroelectric domains} \\ -d_{33} & \text{in negative ferroelectric domains} \end{cases} \tag{8}$$

and ω is the angular frequency of the fundamental beam; k^{ω} and $k^{2\omega}$ are the wavenumbers of the fundamental beam and that of the SHGL, respectively; c is the speed of light in vacuum and d_{33} is the largest nonlinear optical coefficient for $LiNbO_3$ crystal.

After passing through N FC(n) ferroelectric layers, the SHGL electric field E_2 can be given as [12]

$$E_2(N) \approx -\frac{128\pi\omega^2 d_{33} E_1^2}{k^{2\omega}c^2\,\Delta k} \mathrm{e}^{(\mathrm{i}/2)(\pi - \Delta k\, l)} \sin\left(\frac{\Delta k\, l}{2}\right) \mathrm{e}^{\mathrm{i}X_{pq}} \frac{\sin X_{pq}}{X_{pq}} \tag{9}$$

where [14]

$$\Delta k = k^{2\omega} - 2k^{\omega} = \frac{4\pi}{\lambda}\,[n_2(\lambda) - n_1(\lambda)] \tag{10}$$

and where $n_2(\lambda)$ and $n_1(\lambda)$ are the refractive indices of the SHGL and the fundamental beam, respectively, and λ is the wavelength of the fundamental beam. From reference [15] we know that in formula (9)

$$X_{pq} = \frac{\pi\varphi_n}{1 + \varphi_n^2}(q\varphi_n - p) \tag{11}$$

where p and q are integers. Therefore it is obvious that the SHGL intensity peaks are indexed by two integers, p and q, even though the ferroelectric domain structure is 1D. This is different from the case for periodic optical superlattices. For the intense SHGL peaks, we have [12]

$$\Delta k - \frac{2\pi(q + p\varphi_n)}{D} = 0 \tag{12}$$

where

$$D = \varphi_n l_{\mathrm{A}} + l_{\mathrm{B}}. \tag{13}$$

For this $LiNbO_3$ crystal system we have [13] $\omega = 2\pi c/\lambda$ and $k^{2\omega} = 4\pi n_2/\lambda$. We can also rewrite equation (9) as

$$E(2\omega) = -\frac{64\pi^2 l d_{pq} E_1^2}{n_2\lambda} \mathrm{e}^{(\mathrm{i}/2)(\pi - \Delta k\, l)} \mathrm{e}^{\mathrm{i}X_{pq}} \tag{14}$$

where

$$d_{pq} = d_{33}\,\mathrm{sinc}\left(\frac{1}{2}\,\Delta k\, l\right) \mathrm{sinc}(X_{pq}) \tag{15}$$

and where 'sinc' is defined by

$$\mathrm{sinc}(x) = \frac{\sin x}{x}. \tag{16}$$

The power per unit area in a material with index n is [13]

$$S = \frac{cn}{2\pi} E E^*. \tag{17}$$

Thus the input power per unit area of the fundamental beam is

$$s(\omega) = \frac{cn_1}{2\pi} E_1 E_1^*. \tag{18}$$

One can get the output power per unit area of the SHGL as follows by the use of equation (17):

$$s(2\omega) = \frac{cn_2}{2\pi} E_2 E_2^* = \frac{8192\pi^5 l^2 d_{pq}^2 s(\omega)^2}{n_1^2 n_2 c \lambda^2} \tag{19}$$

and thus the efficiency of the output power is

$$\zeta = \left(\frac{s(2\omega)}{s(\omega)}\right)\% = \left(\frac{8192\pi^5 l^2 d_{pq}^2 s(\omega)}{n_1^2 n_2 c \lambda^2}\right)\%. \tag{20}$$

In particular, in this paper, we discuss the properties of the ordinary light (o-light) and extraordinary light (e-light) in the output SHGL for $LiNbO_3$ superlattices.

Substituting equations (10), (13), and (5) into equation (12), one can obtain the formula for the intense SHGL in terms of the wavelength λ in reciprocal space as

$$\left(\frac{1}{\lambda}\right)_{pq} = \frac{q + p\varphi_n}{4[n_2(\lambda) - n_1(\lambda)](1 + \varphi_n) l} \tag{21}$$

where $n_1(\lambda)$ and $n_2(\lambda)$ are functions of λ. For $LiNbO_3$ crystals, the refractive indices of the fundamental beams of o-light and e-light are fitted to the following modified Sellmeier equations [16]:

$$n_{1o}^2 = 4.9048 - \frac{0.117\,68}{0.047\,50 - \lambda^2} - 0.027\,169\,\lambda^2 \tag{22}$$

$$n_{1e}^2 = 4.5820 - \frac{0.099\,169}{0.044\,432 - \lambda^2} - 0.021\,950\,\lambda^2. \tag{23}$$

By means of equation (21), equation (22), and equation (19), we can obtain the relation between the SHGL output power per unit area $s(2\omega)_o$ of the o-light and the wavelength of the fundamental-frequency light λ. Figure 2 shows the result for o-light with $n = 1$, $\varphi_n = \varphi_1 = \tau = (1 + \sqrt{5})/2, c = 3.0 \times 10^8\ \mathrm{m\,s^{-1}}, l = l_0 = \pi/\Delta k_0, \Delta k_0 = 4\pi(n_{20} - n_{10})/\lambda_0$, $\lambda_0 = 1.318 \times 10^{-6}$ m, input power per unit area $s(\omega) = 54\ \mathrm{J\ m^{-2}}$, $n_{10} = 2.1453$, and $n_{20} = 2.1970$.

Similarly, by use of equation (21), equation (23), and equation (19), one can also get the relation between the SHGL output power per unit area $s(2\omega)_e$ of the e-light and the wavelength of the fundamental-frequency light λ. The intense peaks in figure 3 are those of e-light. The initial conditions are the same as those for o-light.

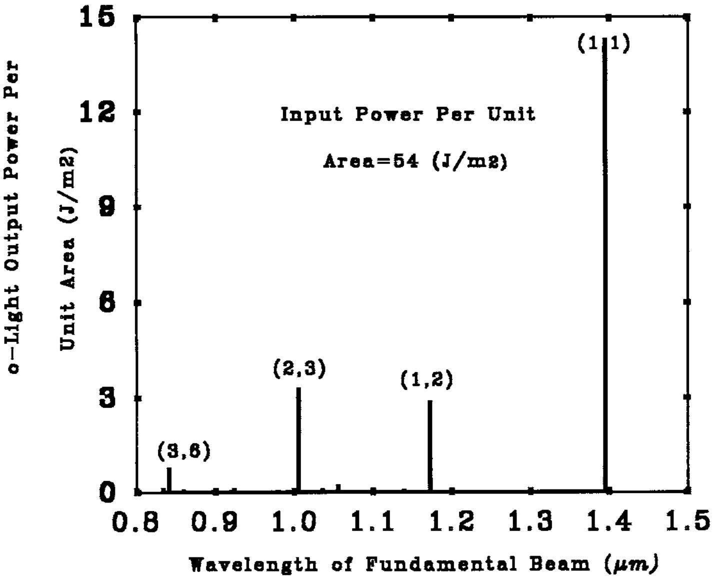

Figure 2. A schematic diagram of the o-light output power per unit area $s(2\omega)_o$ versus the wavelength of the fundamental beam λ for $LiNbO_3$ crystal.

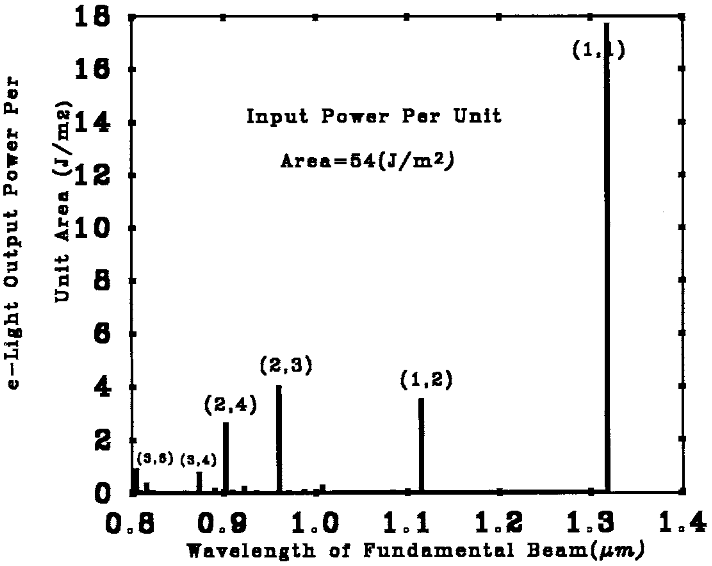

Figure 3. A schematic diagram of the e-light output power per unit area $s(2\omega)_e$ versus the wavelength of the fundamental beam λ for $LiNbO_3$ crystal.

3. The absence effect of the output SHGL spectral lines

From equation (9) it can easily be deduced that some SHGL spectral lines will disappear when $\sin(\Delta k\, l/2) = 0$, i.e.,

$$\frac{\Delta k\, l}{2} = m\pi \tag{24}$$

where m is an integer. Substituting equation (13) and equation (12) into equation (24), one can get

$$q + p\varphi_n = 2m(1 + \varphi_n). \tag{25}$$

For the irrational quadratic number φ_n and the integers q and p, we have

$$q = p = 2m \tag{26}$$

where m is an integer. Therefore the SHGL spectral lines $(2m, 2m)$ will disappear; this is one kind of absence effect for SHGL spectral lines.

On the other hand, from equation (9) it can also be deduced that there is another kind of absence effect for SHGL spectral lines. When $\sin(X_{pq}) = 0$, the corresponding spectral peaks will disappear. Where φ_n is the positive root of the characteristic equation (3), and the value of X_{pq} is given by equation (11), we have

$$p = \frac{qn}{2} \tag{27}$$

i.e.,

$$p = \begin{cases} (2j+1)m & \text{if } n = 2j+1 \text{ and } q = 2m \\ jm & \text{if } n = 2j \text{ and } q = m \end{cases} \tag{28}$$

where j and m are integers. From equation (28) one obtains that the SHGL spectral lines $(2m, (2j+1)m)$ and/or (m, jm) will have disappeared; this is another kind of absence effect for SHGL spectral lines.

4. Brief summary

After describing the method for constructing a class of 1D FC(n) ferroelectric domain systems and after studying the properties of the electric field of the SHGL, we obtained the output power per unit area of the SHGL and the efficiency of the output power versus input power. Then, by means of the formula for the output SHGL, we studied the absence effect for the output SHGL spectral lines. The rules for the disappearance of spectral lines for o-light and e-light were found.

Acknowledgments

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